Ragnar Granit Institute
Tampere University of Technology

71210 Bioelektroniikka - Bioelectromagnetism
Laskuharjoitus 4 – Exercise 4, 6.10.2004

Some units related to the cell and its membrane, as used in Malmivuo & Plonsey

C_m	membrane capacitance per area [F/m^2]
c_m	membrane capacitance per length [F/m]
r_i, r_o	axial resistances per length [Ω/m]
R_i, R_o	axial resistances [Ω]
R_m	membrane resistance [Ωm^2]
r_m	membrane resistance times length [Ωm]

1. The intracellular resistance of a nerve cell is $8.2*10^6$ Ω/cm (r_i). Resistance of the cell membrane is $1.5*10^4$ Ωcm and capacitance 12 nF/cm (c_m). Calculate the characteristic length and time constant of the axon. (start from the general cable equation to see how the time constant is derived) (*answer:* λ *= 428* μ*m,* τ *= 180* μ*s*)

2. The rheobasic current of the nerve cell in the previous exercise is 2 mA.

   a) What is the strength-duration equation of the cell? How long will it take to reach the stimulus threshold with a 2.5 mA stimulus current? What is the chronaxy of the cell? (*answer: 0.29 ms, 125*μ*s*)
   b) Determine the propagation speed of an action pulse if the cell diameter is 100 µm and coefficient K = 10.47 1/ms and in propagation equation

$$\Theta = \sqrt{\frac{\mathrm{K}r}{2\rho C_m}}.$$

   ρ is the intracellular resistivity (*answer: 327 cm/s*).

3. The membrane resistivity of a long cylindrical cell (diameter 80 µm) is 120 Ωcm^2. The resistivity of the cell axoplasm is 1 Ωcm and the extracellular resistance is 3 Ω/cm. The time constant of the cell is 1 ms. What are the characteristic length of the cell and the capacitance of the cell membrane per m^2. (answer: $\lambda = 0.49cm$, $C_m = 8.4\mu F/cm^2$)

4. In a branching neural process the larger branch divides into two smaller ones as shown in Figure 1. In terms of distance along the dendrite, the distance between B and C is the same as the distance between B and D, the distance between C and D, and the distance between A and B.

   a) Current is injected into electrode B, and measured from the C and D. What will be observed? How is the measured data at C as compared at D?

b) Current is injected into electrode C, and measured from electrodes B and D. What will be observed? How is the measured data at B as compared at D?

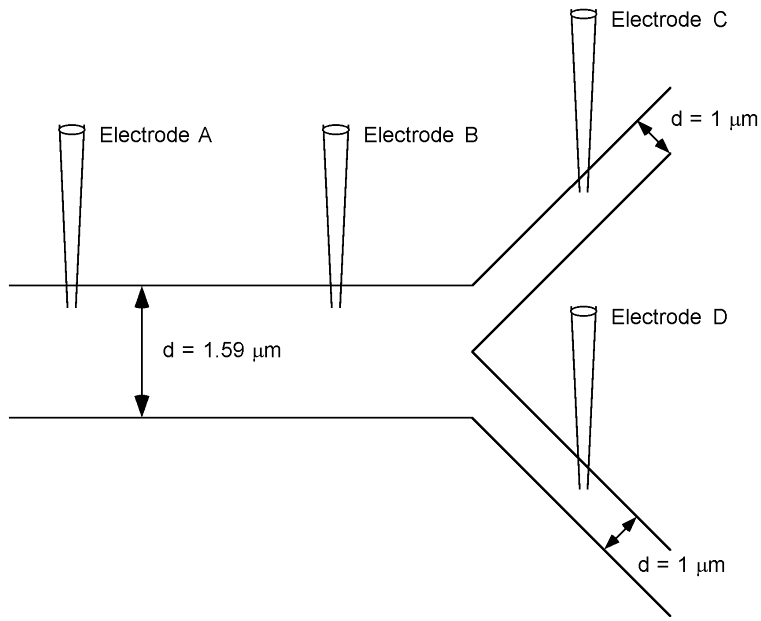

Figure 1. Branching neural system.

5. A current pulse with duration exceeding the membrane time constant by a factor of 10 having amplitude of 0.1 nA is injected into the spherical cell. The membrane potential is measured and observed to rise exponentially to a final value of 3.557 mV. Half maximum voltage response is observed at t = 2.773 ms. The membrane capacitance $C_m = 1\mu F/cm^2$. Calculate the membrane resistance R_m. (answer: $4k\Omega cm^2$)

6. Figure 2 represents an electrical model of a synapse. When impulse arrives to the presynaptic terminal the transmitter (acetylcholine ACH) is released to the synaptic cleft (after 0.5 ms). The cell membrane will become permeable to Na^+ and K^+ ions and the membrane potential tends to shift towards the mean of the Nernst voltages of the Na^+ and K^+ ions (=reversal voltage V_{rev}).

a) What is the resting potential of the presynaptic terminal (ACH-switch open)? (*answer: -90 mV*)
b) What is the reversal voltage, V_{rev} (ACH-switch closed)? (*answer: 20mV*)

C_m (membrane capacitance per area) = 1 $\mu F/cm^2$
Nernst voltages $V_K = -100$ mV and $V_{Na} = 60$ mV
$R_K = 108\ \Omega$, $R_{Na} = 1.5*10^9\ \Omega$, $\Delta R_K = 10^5\ \Omega$, $\Delta R_{Na} = 10^5\ \Omega$.

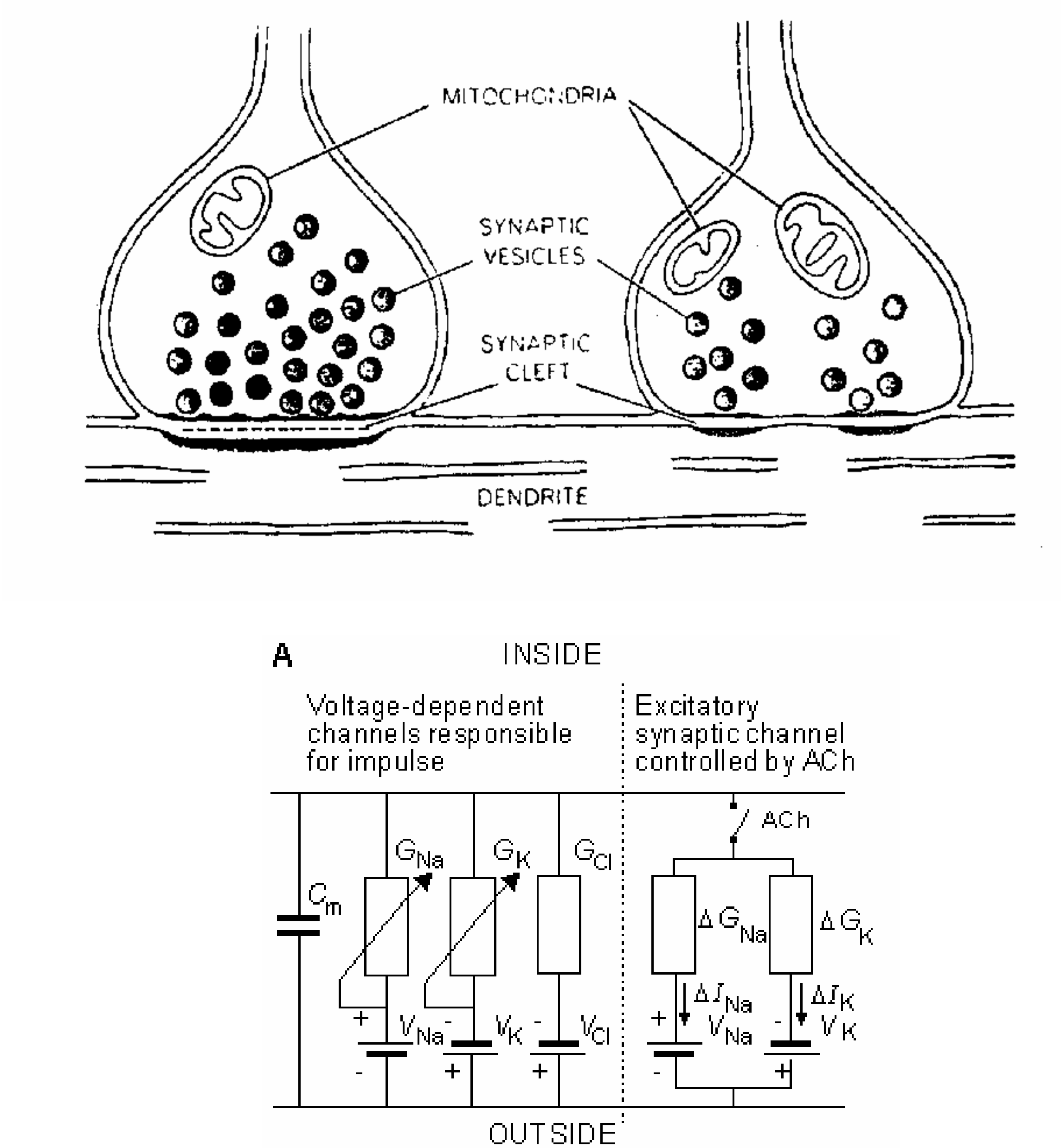

Figure 2. Synapse and its' electrical model.